Selective Message Relaying for Multi-Hopping Vehicular Networks

Bengi Aygun‡, Chung-Wei Lin‡, Shinichi Shiraishi†, and Alexander M. Wyglinski*
*Department of Electrical and Computer Engineering, Worcester Polytechnic Institute, Worcester, MA
†Toyota InfoTechnology Center, Mountain View, CA
Emails: {baygun, alexw}@wpi.edu, {cwlin, sshiraishi}@us.toyota-itc.com

Abstract—Multi-hopping vehicular networks (VANETs) provide environmental awareness across large regions even if there does not exist direct communication between several vehicles within the region. However, the volume of messages that are received and rebroadcasted via the relay vehicles can cause broadcast storms and network jams. In this paper, we propose a selective message relaying algorithm to enable efficient information sharing of multi-hopping VANETs. The proposed algorithm is very general so that it can be applied to many real-world applications. We use a clustering mechanism to group messages that include the same or very similar information, and the number of clusters is adaptively varied based on the proximity of the messages. Supported by the clustering mechanism, a relay vehicle only rebroadcasts a few messages from each cluster. Numerical results show that the proposed algorithm reduces 75% message rebroadcasting but still keeps 98% environmental awareness on average, and it has good scalability which is especially important for future highly-loaded vehicular networks.

I. INTRODUCTION

The National Highway Traffic Safety Administration of United States (NHTSA) announced that connected vehicles will be mandated by 2019, and these vehicles will support both vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications [1]. The key element for enabling most safety applications within vehicular networks (VANETs) is information sharing. By knowing their operating environments, connected vehicles are going to be better equipped for decision-making and traffic efficiency. In order to enable information sharing, connected vehicles use periodic message exchanges, referred to as Basic Safety Messages (BSMs) in the United States and Cooperative Awareness Messages (CAMs) in Europe, that make vehicles aware of their surrounding environments. In addition to cooperation with single-hop neighbors, information can also be transmitted over multiple hops from source to destination in order to increase the environmental awareness.

Multi-hop relaying within VANETs enhances the awareness across a larger region relative to the transmission range [2]. The collision risk caused by inclement weather conditions such as fog, rain, and snow can be significantly reduced by relaying VANETs. Furthermore, the judgment on environment can be delayed so that same (but redundant) decision processes can be avoided. Due to the benefits of multi-hopping VANETs, there are numerous research efforts published in the literature to address the challenges of multi-hopping strategy. Barradi et al. [3] proposed a MAC layer strategy to avoid broadcast storm by adjusting the backoff time in highway scenarios. Hoque and Kwon [4] proposed to choose the packets to rebroadcast based on packet directions. Although this technique helps decrease the network load, a relay vehicle needs to know the types of applications that destination vehicles use. For instance, intersection warning messages from a relay vehicle need to be received by the vehicles behind the relay vehicle. On the other hand, ambulance warning message from a relay vehicle usually need to be received by the vehicles ahead of the relay vehicle. Xiang et al. [5] chose the rebroadcasted messages based on their packet values. The data preference is a promising idea, although the messages are checked one-by-one, which causes processing delay.

To enable safe and efficient multi-hopping vehicular communications, there are several technical challenges associated with VANETs:

- Periodic broadcasting or rebroadcasting results in a significant number of messages.
- With the large number of messages, high network and processing loads cause significant message delays and message losses.
- The existing works require specific information in messages such as message directions.

In this paper, we propose a selective message relaying algorithm that relays information and only rebroadcasts a few non-redundant messages that are useful for other vehicles. To the best of our knowledge, selective message relaying in V2V networks without contextual knowledge of the destination vehicles has not been proposed before. The proposed algorithm rebroadcasts urgent safety messages immediately without any selection. For the rest of the received messages, the proposed algorithm utilizes a hierarchical clustering mechanism to identify similar information among those received messages. The number of clusters is decided by a technique considering the proximity of the messages, and only a few messages from each cluster are selected and rebroadcasted. It is important to note that the proposed algorithm selects messages and rebroadcasts them. It is a completely different approach from traditional routing techniques, such as Position Based Forwarding (PBF), Contention-Based Forwarding (CBF), and Ad hoc On-Demand Distance Vector (AODV), which choose destination vehicles and forward messages.
To address the challenges mentioned above, the main contributions of our paper are:

- We propose an algorithm utilizing a clustering mechanism, detecting redundant information, and selecting messages to rebroadcast without sacrificing environmental awareness.
- The proposed algorithm decreases network and processing loads since only a few messages from each cluster are rebroadcasted.
- The proposed algorithm uses only general information in messages such as time stamp, location, and message priority so that it can be applied to many real-world applications.

The traffic conditions are created in Simulation of Urban Mobility (SUMO) simulation environment [6], and the proposed approach is implemented in the Geometry-based, Efficient Propagation Model for Vehicular Communication (GEMV) simulation software package [7]. Numerical results show that the proposed algorithm reduces 75% message rebroadcasting but still keeps 98% environmental awareness on average, and it has good scalability which is especially important for future highly-loaded vehicular networks.

The rest of the paper is organized as follows. In Section II, we describe the system architecture, several real-world scenarios, and the metrics for the evaluation of multi-hopping mechanisms. In Section III, we describe the proposed selective message relaying algorithm. In Section IV, we discuss experiment results, and several concluding remarks are made in Section V.

II. VEHICULAR ENVIRONMENT

A. System Architecture

In this section, we describe the multi-hopping system architecture for VANETs. Vehicles between the source and destination vehicles play the roles of relay vehicles to propagate information. Figure 1 shows an example. The relay vehicle receives the messages broadcasted by source vehicles near the event region. The received messages are denoted as \( \mu_i \) where \( i = 1, 2, \ldots, M \) and \( M \) is the number of received messages. The relay vehicle decides which information is crucial or worth rebroadcasting and rebroadcasts the selected messages. The rebroadcasted messages are denoted as \( \mu_i' \) where \( i = 1, 2, \ldots, L \) and \( L \) is the number of rebroadcasted messages. We assume that the relay vehicle has sufficient computing ability which is expected in the near future (we will also demonstrate the efficiency of the proposed algorithm in Section IV), or the computation can be supported by other devices such as laptops. We also assume that all vehicles have dedicated short-range communications (DSRC) devices [8], and there are one transmitting and one receiving omnidirectional antennas at each vehicle.

The standardized broadcast scheme is used [8]. The MAC layer performs the traditional Request to Send/Clear to Send (RTS/CTS) mechanism in order to avoid the hidden node problem and broadcast collisions. Additionally, the exponential backoff time is utilized based on the wireless access in vehicular environments (WAVE) standard [9]. The network architecture utilizes the BSM format, as defined in IEEE 802.11p [8], which includes core state information such as location, speed, and brake status, as well as path history and prediction. These messages are typically on the order of 300–400 bytes [10] with a 6 Mbps data rate and a 10 Hz message rate, and transmitted over 300–500 meters. Usually, a vehicle can handle up to 2,000 messages per second, so some works adjust the message rates to avoid congestion. Rather than decreasing the message rates to solve the network congestion problem, our goal in this paper is to let the relay vehicle select crucial or representative messages and rebroadcast them to the destination vehicle(s), so we will focus on the selection algorithm in Section III.

B. Real-World Applications

In this section, we present several real-world applications that the proposed selective message relaying algorithm can be applied. The first application is intersection awareness, as shown in Figure 2(a). The source vehicles near the intersection broadcast the environmental situation. The relay vehicle eliminates the redundant messages and rebroadcasts useful information to the destination vehicle which is approaching the intersection. The second application is emergency vehicle warning, as shown in Figure 2(b). The source vehicles create the messages indicating the presence of the approaching emergency vehicle. The relay vehicle eliminates the redundant messages and rebroadcasts useful information to the destination vehicle ahead so that it can yield the emergency vehicle. The other two applications are side road merging and sharp curve assistant as shown in Figure 2(c) and Figure 2(d). They have several common features. The destination vehicle does not have a direct line-of-sight to the environment where the source vehicles are located. Furthermore, similar (or even redundant) messages are created from different source vehicles so that a reply vehicle should perform message selection. The
Fig. 2. Real-world applications that the proposed selective message relaying algorithm can be applied. S, R, and D represent the source, relay, and destination vehicles, respectively.

application list can be extended to cover collision warning, parking lot assistance, traffic jam warning, and so on.

C. Performance Metrics

To evaluate cooperative awareness in vehicular environments, we use the following four metrics:

- **Rebroadcasting Rate**: The number of messages that are rebroadcasted to the destination vehicle(s). This metric is highly related to processing delay and system efficiency.
- **Processing Delay**: The summation of computing delay and queuing delay. The computing delay is measured directly, and the processing delay is calculated by the M/M/1 queuing model [11]. Assuming a Poisson arrival rate and exponential service time, the expected response time for broadcasted messages is defined as [12]

\[
E[\text{response time}] = \frac{1}{\mu} \left(1 + \frac{\rho}{1 - \rho}\right),
\]

where \(\mu\) is the service rate of the relay vehicle, which is set to 2,000 messages per second, and \(\rho\) is the traffic intensity for first-come-first-serve behavior, which is defined as

\[
\rho = \frac{V \lambda}{\mu},
\]

where \(V\) is the number of source vehicles, and \(\lambda\) is the message rate, which is set to 10 Hz (as defined in the standard). Processing delay measures the system efficiency.

- **Vehicle Coverage**: The number of vehicles that receive the message of corresponding events. This metric measures the level of environmental awareness.
- **Incident Awareness Rate (IAR)**: The number of incidents that the relay vehicle rebroadcasts over the total number of incidents. To find the IAR in this work, we consider incidents for safety, *i.e.*, Access Category (AC) 2 and AC 3 [8], in the 100-meter radius region and in the last 20 seconds, since these incidents have the most urgent priority.

III. PROPOSED ALGORITHM

The goal of the proposed algorithm is to identify crucial or representative messages received by a relay vehicle and only rebroadcast them for the destination vehicles. Due to different environments and traffic conditions, the relay vehicle will receive various numbers of messages at different broadcast periods, so the proposed algorithm uses an adaptive clustering mechanism which can be used in different environments and traffic conditions. The notations which are used throughout this paper is listed in Table I, and the proposed selective message relaying algorithm is summarized in Algorithm 1.

A. Message Preprocessing

Clustering mechanisms find the set of objects that are more similar to each other than the other sets of objects\(^2\) [13]. In this paper, we consider three features to define a message in

\(^2\)Objects are referred to received messages of a relay vehicle in our case.
TABLE I
NOTATION TABLE.

<table>
<thead>
<tr>
<th>V</th>
<th>Number of source vehicles</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Number of received messages at the relay vehicle</td>
</tr>
<tr>
<td>N</td>
<td>Number of features of a received messages</td>
</tr>
<tr>
<td>L</td>
<td>Number of rebroadcasted messages at the relay vehicle</td>
</tr>
<tr>
<td>µi</td>
<td>i-th received message at the relay vehicle</td>
</tr>
<tr>
<td>µi'</td>
<td>i-th rebroadcasted message at the relay vehicle</td>
</tr>
<tr>
<td>χk</td>
<td>k-th feature of a received message</td>
</tr>
<tr>
<td>x_{ih}</td>
<td>Value of χk of µi</td>
</tr>
<tr>
<td>d_{ij}</td>
<td>Distance between µi and µj</td>
</tr>
<tr>
<td>AC</td>
<td>Access Category</td>
</tr>
<tr>
<td>P_i</td>
<td>Message type of µi after preprocessing</td>
</tr>
<tr>
<td>T_i</td>
<td>Temporal feature of µi after preprocessing</td>
</tr>
<tr>
<td>α_i</td>
<td>Mean of exponential of µi (based on message type)</td>
</tr>
<tr>
<td>τ_i</td>
<td>Time that µi is created at the source vehicle</td>
</tr>
<tr>
<td>τ_i'</td>
<td>Time that µi is processed at the relay vehicle</td>
</tr>
<tr>
<td>S_i</td>
<td>Spatial feature of µi after preprocessing</td>
</tr>
<tr>
<td>δ_i</td>
<td>Distance between the source and relay vehicles</td>
</tr>
<tr>
<td>AR</td>
<td>Awareness Range</td>
</tr>
</tbody>
</table>

ALGORITHM 1: Selective Message Relaying Algorithm

1: /* Message Preprocessing */
2: for each message µi do
3:   if Message Type = AC3 then
4:     Rebroadcast;
5:   else
6:     P_i = 1/16, 1/4, 1 for AC0, AC1, AC2, respectively;
7:     T_i = \frac{\alpha_i}{\tau_i} (t - \tau_i);
8:     if D_i ≤ AR then
9:       S_i = 1 - D_i / AR;
10:  else
11:     S_i = 0;
12: end if
13: end if
14: end for
15: /* Message Clustering */
16: Compute the distances between messages;
17: Build hierarchy;
18: Decide the number of clusters;
19: Decide the members of clusters;
20: /* Message Selection for Rebroadcasting */
21: Rebroadcast the closest messages to the centroids of clusters;

the clustering space: message type, time stamp, and location that the message is created. Although these three features are sufficient to define the similarity of messages, the interval (difference between the maximum and minimum values) of these features are relatively different from each other. For example, the distance between the source and relay vehicles is [0, 300] meters, while the time stamp is [0, 60] seconds. To develop a reliable clustering mechanism, we need to process the received messages first and model the features in similar intervals. The modeling approach of each feature is addressed below. Note that we define the proposed algorithm with these three features to demonstrate the general idea. Besides them, any information in BSM can be utilized as a feature for some specific applications.

1) Message Type: The events that the broadcasted message include have different priority levels. In IEEE 802.11p standard [8], four different ACs, i.e., message priorities, are defined. The first category, AC0, is assigned to the lowest priority messages, which are non-safety and non-urgent applications. The second category, AC1, is assigned to non-urgent events. The third category, AC2, is for environmental awareness or presence of other vehicles, especially when drivers have limited vision abilities. The highest priority, AC3, defines urgent safety messages [14].

With the message types, we assign the weights (P_i) for messages with AC0, AC1, AC2 to 1/16, 1/4, 1, respectively. All urgent safety messages, i.e., AC3, are rebroadcasted by default, so they do not need any message selection or weight assignment.

2) Temporal Feature: Each broadcasted message includes the time stamp defined in Coordinated Universal Time (UTC) format. A relay vehicle computes the difference between the times that a message is created and the current time. We assign the temporal feature for µi as [5]

\[ T_i = e^{-\frac{\tau_i - \tau_i'}{\alpha_i}} \]

where \( \tau_i' \) is the time that \( \mu_i \) is processed at the relay vehicle (all messages should be regarded as being processed at the same time), \( \tau_i \) is the time that \( \mu_i \) is created at the source vehicle, and \( \alpha_i \) is the mean of exponential, which is defined based on message type. Here, we assign the means of AC0, AC1, AC2 to 4, 8, 16, respectively. Again, AC3 does not need this assignment.

Exponential modeling provides the property that a newer message has a higher value and an older one has a lower but never non-zero value (a message may still be useful for the destination vehicle although it gets older). A unique mean value is defined for each message type since a message with higher priority should decay slower as time goes. On the other hand, a message with lower priority should decay faster. For example, if two messages are created at the same time, one with AC0 decays faster than the other with AC2.
S / R / D: Source / Relay / Destination Vehicle

ϕ
R
ϕ
S0
S1
S1 is closer than R;
D cannot benefit from relaying message from R
R is closer than S1;
S1 is closer than S0
R is closer than S1;
S0 is closer than S1

S / R / D: Source / Relay / Destination Vehicle

by message relaying. Assuming r
sages, where the later one has a larger covered area increased
between selecting closer messages and selecting further mes-

3) Spatial Feature: The relevance of messages received
by a relay vehicle has a negative correlation to the distance
between the source and relay vehicles. We assign the spatial
feature for \( \mu_i \) as

\[
S_i = \begin{cases} 
1 - \frac{\delta_i}{AR}, & \text{if } \delta_i \leq AR; \\
0, & \text{otherwise},
\end{cases}
\]

(4)

where \( \delta_i \) is the distance between the source and relay vehicles and AR is the awareness range that is pre-defined. The setting here considers messages with shorter distances as more important messages (less probability to be clustered with other messages). It is based on the observation that the source vehicles of those messages are usually closer to destination vehicles which can benefit from message relaying, which is illustrated in Figure 4.

However, it should also be mentioned that there is a trade-off between selecting closer messages and selecting further messages, where the later one has a larger covered area increased by message relaying. Assuming \( r \) as the transmission range of a vehicle and \( u \) as \( \frac{5}{7} \), the covered area increased by message relaying can be estimated as \( r^2 \left( 0.14 + 2.01u + 0.14u^3 \right) \) (the analysis is attached in the appendix). Its expected value is around \( 1.18r^2 \) when \( u \) is uniformly distributed in \([0, 1]\) and around \( 1.54r^2 \) when the selected messages are uniformly distributed in the circle of the transmission range. System designers can consider different design metrics, together with the corresponding applications and message clustering, during design stages. In the following sections, we will focus on message clustering as a general solution to reduce the number of rebroadcasted messages.

The preprocessing above provides the value of each feature needed for clustering message, and each feature is in the interval of \([0, 1]\). As an example, the raw data shown in Figure 3(a) is transformed to the version as shown in Figure 3(b). The clustering is then performed on the features after preprocessing.

### B. Message Clustering

There are three main clustering methods that are commonly used: \( K \)-means [15], density-based [16], and hierarchical [17]. \( K \)-means clustering associates each cluster with its centroid. \( K \)-means clustering is not the best for the selective message relaying problem since its performance is very dependent on initial cluster centroids and the value of \( K \). Furthermore, outlier data points, i.e., those very different messages from the others, may cause \( K \)-means clustering to be nonfunctional.

Regarding density-based clustering, the data set is evaluated by clustering high-density data points and leaving the low-density data points out of the clusters as outliers. This does not fit our problem since message clustering considers the similarities of the messages instead of how they are distributed. On the other hand, hierarchical clustering is independent of the initial centroids and capable of adapting to various information sets. Therefore, we use the agglomerative hierarchical clustering as our clustering mechanism.

In hierarchical clustering, each object is initially a cluster and then the closest two clusters are merged until a single cluster remains. The main paradigm of hierarchical clustering is the proximity matrix. Different approaches are proposed to define the distance between clusters such as the minimum or maximum distance between clusters. In this work, we use the average distance between clusters since it is more robust to outliers and noise. The Euclidean distance between message \( i \) and message \( j \) can be computed as

\[
d_{ij} = \sqrt{\sum_{k=1}^{N} (x_{ik} - x_{jk})^2},
\]

(5)

where \( N \) is the number of features of a message and \( x_{ik} \) is the \( k \)-th feature of \( \mu_i \). A set of \( M \) received messages is illustrated in Figure 3(a).

We use the \( L \)-method to obtain the number of clusters [18]. The method to obtain the adaptive number of clusters is based...
Fig. 7. Experimental traffic data. The environment map is created using Open Street Map [20]. The buildings in the chosen area are defined by white blocks. Vehicle traffic, illustrated as red vehicles, is created using SUMO [6] based on the environmental map. The random traffic defined within the area is used as an input to the GEMV$^2$ simulator [7]. The link colors between vehicles show the link powers. If the link color is dark blue, the channel is noisy and experiencing strong fading. If the link color is red, the channel has little noise and fading.

on the distance between messages in the clustering space [19]. The distance set is divided into two subsets, and the curve fitting is performed for these two subsets. The intersection of the extensions of the two fitting lines is decided as the number of clusters. Figure 5 illustrates the method, and the number of clusters is detected as 9. This number is used in the hierarchy of messages as shown in Figure 6, where the bottom part of the hierarchy is omitted. As a result, the member of clusters are decided as the corresponding branches below the red line.

C. Message Selection for Rebroadcasting

The received messages at a relay vehicle are clustered based on their distances to each other in the clustering space. We use the centroids of clusters to represent the corresponding clusters, and the closest messages to the centroids are selected to be rebroadcasted. It should be noted that the rebroadcasted messages are with their features before preprocessing, where their features after preprocessing are only for clustering. Dependong on the requirement of application context, multiple messages from each cluster can be selected to be rebroadcasted.

IV. NUMERICAL RESULTS

A. Simulation Setup

We analyze the performance of the selective message relaying algorithm within a VANET environment using the GEMV$^2$ Vehicle-to-X (V2X) propagation simulator and MATLAB [7]. GEMV$^2$ is a computationally efficient propagation model for V2X communications, which explicitly accounts for surrounding objects. The experimental traffic data is created in SUMO for a 1 km$^2$ region (shown in Figure 7) and used as an input to GEMV$^2$. The link colors between vehicles highlight the link powers. The experiment setup is summarized in Table II. Experiment is run with Intel Core i7 and 2.2 GHz processor, and the selective message relaying algorithm takes less than 2 msec. Although current in-vehicle processors are not so powerful, we assume that a relay vehicle has sufficient computing ability which is expected in the near future, or the computation can be supported by other devices such as laptops.

B. Results

In Figure 8, the messages which are created in the transmission range around the relay vehicle are shown. The colors of messages are referred to the priorities of messages, and the messages with grey stars are selected to be rebroadcasted.

In Figure 9(a), the number of rebroadcasted messages (rebroadcasting rate) is shown with respect to the number of connected vehicles. The data preference mechanism, packet-valuecast (PVcast), proposed by Xiang et al. [5] is used as a comparative approach. Compared with rebroadcasting without any selection, the proposed algorithm decreases the number of rebroadcasted messages by around 75%. Besides, it also rebroadcasts fewer messages than PVcast.

In Figure 9(b), the processing delay is shown with respect to the number of connected vehicles. In the cases of fewer vehicles, the proposed algorithm has the same or slightly higher processing delay than the two comparative approaches. This is due to the computation time of the proposed algorithm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Message Size (Byte)</td>
<td>375</td>
</tr>
<tr>
<td>Message Rate (Hz)</td>
<td>10</td>
</tr>
<tr>
<td>Transmission Power (dBm)</td>
<td>23</td>
</tr>
<tr>
<td>Carrier Sense Threshold (dBm)</td>
<td>-90</td>
</tr>
<tr>
<td>Noise Floor (dBm)</td>
<td>-113</td>
</tr>
<tr>
<td>Max Transmission Range (m)</td>
<td>300</td>
</tr>
<tr>
<td>Message Period (msec)</td>
<td>100</td>
</tr>
</tbody>
</table>

Fig. 8. Message map. The points show the locations where the messages are created. The colors of messages are referred to the priorities of messages, and the messages with grey stars are selected to be rebroadcasted.

TABLE II
SYSTEM PARAMETER SETUP
considered by system designers while choosing the weights. These effects should be more messages in the environment. These effects should be more important when there are more message types, which is more important when there are more connected vehicles. This is also because it provides better differentiation for different ACs and cause less incident awareness. Another result showed that the proposed algorithm reduces 75% message rebroadcasting but still keeps 98% environmental awareness on average, and it has good scalability which is especially important for future highly-loaded vehicular networks. Future directions include specialization for applications, trade-off analysis between design metrics (such as performance and coverage), and design space exploration and optimization.

### V. Conclusion

In this paper, we proposed a selective message relaying algorithm for multi-hopping vehicular networks. The proposed algorithm assigns features to messages and applies a clustering mechanism to group messages so that a relay vehicle only needs to rebroadcast a few messages from each cluster. Numerical results showed that the proposed algorithm reduces 75% message rebroadcasting but still keeps 98% environmental awareness on average, and it has good scalability which is especially important for future highly-loaded vehicular networks. Future directions include specialization for applications, trade-off analysis between design metrics (such as performance and coverage), and design space exploration and optimization.

### Appendix

This appendix is to estimate the covered area increased by message relaying. System designers can consider it, together with the corresponding applications and the proposed algorithm in this paper, during design stages. The following derivation considers a basic scenario as shown in Figure 10(a). Assuming that $\delta$ is the distance between the source and relay vehicles and $r$ is the transmission range of a vehicle, we calculate $\theta$ and define $u$ as follows:

$$\cos \theta = \frac{\delta}{2r},$$  \hspace{1cm} (6)

$$\theta = \arccos \left( \frac{\delta}{2r} \right),$$  \hspace{1cm} (7)

$$u = \frac{\delta}{r},$$  \hspace{1cm} (8)
where \( u \) is in \([0, 1]\). The following approximations based on the Taylor series will be used:

\[
\begin{align*}
\cos x & \approx 1 - \frac{x^2}{2}, \\
\arccos x & \approx \sqrt{2 - 2x}, \\
\sqrt{1 + x} & \approx 1 + \frac{x}{2}, \\
\sqrt{3 + x} & \approx \sqrt{3} + \frac{x}{2\sqrt{3}}.
\end{align*}
\]

As shown in Figure 10(b), the covered area increased by message relaying, \( A \), is approximated as follows:\(^3\)

\[
A = A_1 - 2A_2 + A_3
\approx \pi r^2 - 2\theta r^2 + 4 \sqrt{r^2 - \frac{\delta^2}{4}}
\approx \pi r^2 - 2\arccos \left( \frac{\delta}{2r} \right) r^2 + \frac{\delta}{2} \sqrt{4r^2 - \delta^2}
\approx \pi r^2 - 2r^2 \sqrt{2 - \frac{\delta}{r} + \frac{\delta}{2} \sqrt{4r^2 - \delta^2}}
\approx \pi r^2 - 2r^2 \sqrt{2 - \frac{\delta}{r} + \frac{\delta}{2} \sqrt{4r^2 - \delta^2}}
\approx r^2 \left( \pi - 2\sqrt{2 - \frac{\delta}{r} + \frac{\delta}{2} \sqrt{4r^2 - \delta^2}} \right)
\approx r^2 \left( \pi - 2\sqrt{2 - \left( 1 - \frac{\delta}{r} \right) + \frac{\delta}{2} \sqrt{4r^2 - \delta^2}} \right)
\approx r^2 \left( \pi - 2\sqrt{2 - \left( 1 - \frac{\delta}{r} \right) + \frac{\delta}{2} \sqrt{4r^2 - \delta^2}} \right)
\approx r^2 \left( \pi - 3 + \frac{7\sqrt{3}u}{12} - \frac{3u^3}{4\sqrt{3}} \right)
\approx r^2 \left( 0.14 + 2.01u + 0.14u^3 \right),
\]

and the expected value can be computed as:

\[
E[A] \approx \int_0^1 \left( r^2 \left( 0.14 + 2.01u + 0.14u^3 \right) p(u) \right) du,
\]

where \( p(u) \) is probability density function of \( u \) over \([0, 1]\).

The expected value is around \(1.18r^2\) when \( u \) is uniformly distributed in \([0, 1]\) and around \(1.54r^2\) when the selected messages are uniformly distributed in the circle of the transmission range. System designers can have different clustering heuristics with different probability density functions to consider the trade-offs between design metrics.

**References**


